Cognitive Informatics and Denotational Mathematical Means for Brain Informatics

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Abstract. Cognitive informatics studies the natural intelligence and the brain from a theoretical and a computational approach, which rigorously explains the mechanisms of the brain by a fundamental theory known as abstract intelligence, and formally models the brain by contemporary denotational mathematics. This paper, as an extended summary of the invited keynote presented in AMT-BI 2010, describes the interplay of cognitive informatics, abstract intelligence, denotational mathematics, brain informatics, and computational intelligence. Some of the theoretical foundations for brain informatics developed in cognitive informatics are elaborated. A key notion recognized in recent studies in cognitive informatics is that the root and profound objective in natural, abstract, and artificial intelligence in general, and in cognitive informatics and brain informatics in particular, is to seek suitable mathematical means for their special needs that were missing in the last six decades. A layered reference model of the brain and a set of cognitive processes of the mind are systematically developed towards the exploration of the theoretical framework of brain informatics. The current methodologies for brain studies are reviewed and their strengths and weaknesses are analyzed. A wide range of applications of cognitive informatics and denotational mathematics are recognized in brain informatics toward the implementation of highly intelligent systems such as world-wide wisdom (WWW+), cognitive knowledge search engines, autonomous learning machines, and cognitive robots.

Keywords: *Cognitive informatics, abstract intelligence, brain informatics, cognitive computing, cognitive computers, natural intelligence, artificial intelligence, machinable intelligence, computational intelligence, denotational mathematics, concept algebra, system algebra, RTPA, visual semantic algebra, granular algebra, eBrain, engineering applications*

1 Introduction

The contemporary wonder of sciences and engineering has recently refocused on the starting point of them: how the brain processes internal and external information autonomously and cognitively rather than imperatively as those of conventional computers. The latest advances and engineering applications of CI have led to the emergence of *cognitive computing* and the development of *cognitive computers* that perceive, learn, and reason [9, 18, 20, 23, 24, 32]. CI has also fundamentally contributed to autonomous agent systems and cognitive robots. A wide range of applications of CI are identified such as in the development of cognitive computers, cognitive robots, cognitive agent systems, cognitive search engines, cognitive learning systems, and artificial brains. The work in CI may also lead to a fundamental solution to computational linguistics, Computing with Natural Language (CNL), and Computing with Words (CWW) [34, 35].

Cognitive Informatics is a term coined by Wang in the first IEEE International Conference on Cognitive Informatics (ICCI 2002) [6]. Cognitive informatics [6, 8, 11, 12, 26, 27, 28, 29, 31] studies the natural intelligence and the brain from a theoretical and a computational approach, which rigorously explains the mechanisms of the brain by a fundamental theory known as *abstract intelligence*, and formally models the brain by contemporary *denotational mathematics* such as *concept algebra* [Wang, 2008b], *real-time process algebra* (RTPA) [7, 16], system algebra [15, 30], and *visual semantic algebra* (VSA) [19]. The latest advances in CI have led to a systematic solution for explaining brain informatics and the future generation of intelligent computers.

A key notion recognized in recent studies in cognitive informatics is that the root and profound objective in natural, abstract, and artificial intelligence in general, and in cognitive informatics and brain informatics in particular, is to seek suitable mathematical means for their special needs, which were missing in the last six decades. This is a general need and requirement for searching the metamethodology in any discipline particularly those of emerging fields where no suitable mathematics has been developed or of traditional fields where persistent hard problems have been unsolved efficiently or completely [1, 2, 4, 13].

This paper is an extended summary of the invited keynote lecture presented in the 2010 joint *International Conferences on Active Media Technology and Brain Informatics* (AMT-BI 2010), which covers some of the theoretical foundations of brain informatics (BI) developed in cognitive informatics and denotational mathematics. In this paper, cognitive informatics as the science of abstract intelligence and cognitive computing is briefly described in Section 2. The fundamental theories and expressive tools for cognitive informatics, brain Informatics, and computational intelligence, collectively known as denotational mathematics, are introduced in Section 3. Applications of cognitive informatics and denotational mathematics in BI and cognitive computing are elaborated in Sections 4, where the layered reference model of the brain and a set of cognitive processes of the mind are systematically modeled towards the exploration of the theoretical framework of brain informatics.

2 Cognitive Informatics: The Science of Abstract Intelligence and Computational Intelligence

Information is the third essence of the word supplementing energy and matter. A key discovery in information science is the basic unit of information, bit, abbreviated from a "binary digit", which forms a shared foundation of computer science and informatics.

The science of information, informatics, has gone through three generations of evolution, known as the classic, modern, and cognitive informatics, since Shannon proposed the classic notion of information [5]. The classical information theory founded by Shannon (1948) defined *information* as a probabilistic measure of the variability of message that can be obtained from a message source. Along with the development in computer science and in the IT industry, the domain of informatics has been dramatically extended in the last few decades. This led to the modern informatics that treats information as entities of messages rather than a probabilistic measurement of the variability of messages as in that of the classic information theory. The new perception of information is found better to explain the theories in computer science and practices in the IT industry. However, both classic and modern views on information are only focused on external information. The real sources and destinations of information, the human brains, are often overlooked. This leads to the third generation of informatics, cognitive informatics, which focuses on the nature of information in the brain, such as information acquisition, memory, categorization, retrieve, generation, representation, and communication. Information in cognitive informatics is defined as the abstract artifacts and their relations that can be modeled, processed, stored and processed by human brains. Cognitive informatics [6, 8, 11, 12, 26, 27, 28, 29, 31] is emerged and developed based on the multidisciplinary research in cognitive science, computing science, information science, abstract intelligence, and denotational mathematics since the inauguration of the 1st IEEE ICCI'02 [6].

Definition 1. *Cognitive informatics* (CI) is a transdisciplinary enquiry of computer science, information science, cognitive science, and intelligence science that investigates into the internal information processing mechanisms and processes of the brain and natural intelligence, as well as their engineering applications in cognitive computing.

CI is a cutting-edge and multidisciplinary research area that tackles the fundamental problems shared by modern informatics, computation, software engineering, AI, cybernetics, cognitive science, neuropsychology, medical science, philosophy, linguistics, brain sciences, and many others. The development and the cross fertilization among the aforementioned science and engineering disciplines have led to a whole range of extremely interesting new research areas.

The theoretical framework of CI encompasses four main areas of basic and applied research [11] such as: a) fundamental theories of natural intelligence; b) abstract intelligence; c) denotational mathematics; and d) cognitive computing. These areas of CI are elaborated in the following subsections. Fundamental theories developed in CI covers the Information-Matter-Energy (IME) model [8], the Layered Reference Model of the Brain (LRMB) [28], the Object-Attribute-Relation (OAR) model of information/knowledge representation in the brain [12], the cognitive informatics model of the brain [23, 26], Natural Intelligence (NI) [8], and neuroinformatics [12]. Recent studies on LRMB in cognitive informatics reveal an entire set of cognitive functions of the brain and their cognitive process models, which explain the functional mechanisms of the natural intelligence with 43 cognitive processes at seven layers known as the sensation, memory, perception, action, meta-cognitive, meta-inference, and higher cognitive layers [28].

Definition 2. *Abstract intelligence* (αI) is a universal mathematical form of intelligence that transfers information into actions and behaviors.

The studies on αI form a field of enquiry for both natural and artificial intelligence at the reductive levels of neural, cognitive, functional, and logical from the bottom up [17]. The paradigms of αI are such as natural, artificial, machinable, and computational intelligence. The studies in CI and αI lay a theoretical foundation toward revealing the basic mechanisms of different forms of intelligence [25]. As a result, cognitive computers may be developed, which are characterized as a knowledge processor beyond those of data processors in conventional computing.

Definition 3. *Cognitive Computing* (CC) is an emerging paradigm of intelligent computing methodologies and systems that implements computational intelligence by autonomous inferences and perceptions mimicking the mechanisms of the brain.

CC is emerged and developed based on the transdisciplinary research in cognitive informatics and abstract intelligence. The term computing in a narrow sense is an application of computers to solve a given problem by imperative instructions; while in a broad sense, it is a process to implement the instructive intelligence by a system that transfers a set of given information or instructions into expected intelligent behaviors.

The *essences* of computing are both its *data objects* and their predefined computational *operations*. From these facets, different computing paradigms may be comparatively analyzed as follows:

a) Conventional computing
- *Data objects*: abstract bits and structured data
- *Operations*: logic, arithmetic, and functional operations (1a)

b) Cognitive computing (CC)
- *Data objects*: words, concepts, syntax, and semantics
- *Basic operations*: syntactic analyses and semantic analyses
- *Advanced operations*: concept formulation, knowledge representation, comprehension, learning, inferences, and causal analyses (1b)

The latest advances in cognitive informatics, abstract intelligence, and denotational mathematics have led to a systematic solution for the future generation of intelligent computers known as cognitive computers [9, 18].

Definition 4. A cognitive computer (cC) is an intelligent computer for knowledge processing that perceive, learn, and reason.

As that of a conventional von Neumann computers for data processing, cCs are designed to embody *machinable intelligence* such as computational inferences, causal analyses, knowledge manipulations, learning, and problem solving. According to the above analyses, a cC is driven by a *cognitive CPU* with a cognitive learning engine and formal inference engine for intelligent operations on abstract concepts as the basic unit of human knowledge. cCs are designed based on contemporary denotational mathematics [13, 21], particularly concept algebra, as that of Boolean algebra for the conventional von Neumann architecture computers. cC is an important extension of conventional computing in both data objects modeling capabilities and their advanced operations at the abstract level of concept beyond bits. Therefore, cC is an intelligent knowledge processor that is much closer to the capability of human brains thinking at the level of concepts rather than bits. It is recognized that the basic unit of human knowledge in natural language representation is a concept rather than a word [14], because the former conveys the structured semantics of the latter with its intention (attributes), extension (objects), and relations to other concepts in the context of a knowledge network.

Main applications of the fundamental theories and technologies of CI can be divided into two categories. The first category of applications uses informatics and computing techniques to investigate intelligence science, cognitive science, and knowledge science problems, such as abstract intelligence, memory, learning, and reasoning. The second category includes the areas that use cognitive informatics theories to investigate problems in informatics, computing, software engineering, knowledge engineering, and computational intelligence. CI focuses on the nature of information processing in the brain, such as information acquisition, representation, memory, retrieval, creation, and communication. Via the interdisciplinary approach and with the support of modern information and neuroscience technologies, intelligent mechanisms of the brain and cognitive processes of the mind may be systematically explored [33] within the framework of CI.

3 Denotational Mathematics: A Metamethodology for Cognitive Informatics, Brain Informatics, Cognitive Computing, and Computational Intelligence

It is recognized that the maturity of a scientific discipline is characterized by the maturity of its mathematical (meta-methodological) means. A key notion recognized in recent studies in cognitive informatics and computational intelligence is that the root and profound problem in natural, abstract, and artificial intelligence in general, and in cognitive informatics and brain informatics in particular, is to seek suitable mathematical means for their special needs. This is a general need and requirement for searching the metamethodology in any discipline particularly the emerging fields where no suitable mathematics has been developed and the traditional fields where persistent hard problems have been unsolved efficiently or completely [1, 2, 3, 4, 10, 13].

Definition 5. *Denotational mathematics* (DM) is a category of expressive mathematical structures that deals with high-level mathematical entities beyond numbers and sets, such as abstract objects, complex relations, perceptual information, abstract concepts, knowledge, intelligent behaviors, behavioral processes, inferences, and systems.

A number of DMs have been created and developed [13, 21] such as concept algebra [14], system algebra [15, 30], real-time process algebra (RTPA) [7, 16], granular algebra [22], visual semantic algebra (VSA) [19], and formal causal inference methodologies. As summarized in Table 1 with their structures, mathematical entities, algebraic operations, and usages, the set of DMs provide a coherent set of contemporary mathematical means and explicit expressive power for CI, αI, CC, AI, and computational intelligence.

Table 1. Paradigms of Denotational Mathematics

Paradigm	Structure	Mathematical entities	Algebraic operations	Usage
Concept algebra (CA)	$CA \triangleq (C, OP, \Theta) = (\{O, A, R^c, R^i, R^o\}, \{\bullet_r, \bullet_c\}, \Theta_C)$	$c \triangleq (O, A, R^c, R^i, R^o)$	$\bullet_r \triangleq \{\leftrightarrow, \nleftrightarrow, \prec, \succ, =, \cong, \sim, \triangleq\}$ $\bullet_c \triangleq \{\Rightarrow, \overset{-}{\Rightarrow}, \overset{+}{\Rightarrow}, \overset{\sim}{\Rightarrow}, \uplus, \pitchfork, \Lleftarrow, \vdash, \mapsto\}$	Algebraic manipulations on abstract concepts
System algebra (SA)	$SA \triangleq (S, OP, \Theta) = (\{C, R^c, R^i, R^o, B, \Omega\}, \{\bullet_r, \bullet_c\}, \Theta)$	$S \triangleq (C, R^c, R^i, R^o, B, \Omega, \Theta)$	$\bullet_r \triangleq \{\nleftrightarrow, \leftrightarrow, \prod, =, \sqsubseteq, \sqsupseteq\}$ $\bullet_c \triangleq \{\Rightarrow, \overset{-}{\Rightarrow}, \overset{+}{\Rightarrow}, \overset{\sim}{\Rightarrow}, \boxminus, \uplus, \pitchfork, \Lleftarrow, \vdash\}$	Algebraic manipulations on abstract systems
Real-time process algebra (RTPA)	$RTPA \triangleq (\mathfrak{T}, \mathfrak{P}, \mathfrak{N})$	$\mathfrak{P} \triangleq \{:=, \blacklozenge, \Rightarrow, \Leftarrow, \nLeftarrow, \gtrdot, \lessdot, \vert\gtrdot, \vert\lessdot, @, \triangleq, \uparrow, \downarrow, !, \otimes, \boxtimes, \S\}$ $\mathfrak{T} \triangleq \{\mathrm{N, Z, R, S, BL, B, H, P, TI, D, DT, RT, ST}, @e\mathrm{S}, @t\mathrm{TM}, @int\odot, \circledS s\mathrm{BL}\}$	$\mathfrak{R} \triangleq \{\rightarrow, \curvearrowright, \vert, \vert\ldots\vert\ldots, R^*, R^+, R^i, \circlearrowleft, \rightarrowtail, \parallel, \oiint, \vert\vert\vert, \gg, \lightning, \hookrightarrow_t, \hookrightarrow_e, \hookrightarrow_i\}$	Algebraic manipulations on abstract processes
Visual semantic algebra (VSA)	$VSA \triangleq (O, \bullet_{VSA})$	$O \triangleq \{H \cup S \cup F \cup L\}$	$\bullet_{VSA} \triangleq \{\uparrow, \downarrow, \leftarrow, \rightarrow, \odot, \otimes, \boxplus, \measuredangle, @(p), @(\mathrm{x,y,x}), \curvearrowright, \mapsto, \underset{i\mathrm{N}=0}{\overset{n-1\mathrm{N}}{R}}(A_i \mapsto A_{i+1})\}$	Algebraic manipulations on abstract visual objects/patterns
Granular algebra (GrA)	$GA \triangleq (G, \bullet_r, \bullet_p, \bullet_c) = ((C, R^c, R^i, R^o, B, \Omega), \bullet_r, \bullet_p, \bullet_c)$	$G \triangleq (C, R^c, R^i, R^o, B, \Omega, \Theta)$	$\bullet_r \triangleq \{\nleftrightarrow, \leftrightarrow, \prod, =, \sqsubseteq, \sqsupseteq\}$ $\bullet_p \triangleq \{\Rightarrow, \overset{-}{\Rightarrow}, \overset{+}{\Rightarrow}, \overset{\sim}{\Rightarrow}\}$ $\bullet_c \triangleq \{\uplus, \pitchfork, \Lleftarrow\}$	Algebraic manipulations on abstract granules

Among the above collection of denotational mathematics, concept algebra is an abstract mathematical structure for the formal treatment of concepts as the basic unit of human reasoning and their algebraic relations, operations, and associative rules for composing complex concepts. It is noteworthy that, according to concept algebra, although the semantics of words may be ambiguity, the semantics of concept is always unique and precise in CC.

Example 1. The word, "bank", is ambiguity because it may be a notion of a financial institution, a geographic location of raised ground of a river/lake, and/or a storage of something. However, the three distinguished concepts related to "bank", i.e., b_o = bank(organization), b_r = bank(river), and b_s = bank(storage), are precisely unique, which can be formally described in concept algebra [14] for CC as shown in Fig. 1, where $\mathfrak{K}$ represents the entire concepts existed in the analyser's knowledge.

All given concrete concepts share a generic framework, known as the universal abstract concept as modeled in concept algebra as given below.

Definition 6. An *abstract concept*, c, is a 5-tuple, i.e.:

$$c \triangleq (O, A, R^c, R^i, R^o) \qquad (2)$$

where

- O is a nonempty set of objects of the concept, $O = \{o_1, o_2, \ldots, o_m\} \subseteq Þ\mathcal{O}$, where $Þ\mathcal{O}$ denotes a power set of abstract objects in the universal discourse $\mathfrak{U}$.
- A is a nonempty set of attributes, $A = \{a_1, a_2, \ldots, a_n\} \subseteq Þ\mathcal{A}$, where $Þ\mathcal{A}$ denotes a power set of attributes in $\mathfrak{U}$.
- $R^c = O \times A$ is a set of internal relations.
- $R^i \subseteq C' \times c$ is a set of input relations, where C′ is a set of external concepts in $\mathfrak{U}$.
- $R^o \subseteq c \times C'$ is a set of output relations.

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b_oST ≜ (A, O, R^c, R^i, R^o)                    // bank(organization)
      = ( b_oST.A  = {organization, company, financial business,
                      money, deposit, withdraw, invest, exchange},
          b_oST.O  = {international bank, national bank, local bank,
                      investment bank, ATM}
          b_oST.R^c = O × A,
          b_oST.R^i = 𝔎 × b_oST,
          b_oST.R^o = b_oST × 𝔎
        )

b_rST ≜ (A, O, R^c, R^i, R^o)                    // bank(river)
      = ( b_rST.A  = {sides of a river, raised ground, a pile of earth, location},
          b_rST.O  = {river bank, lake bank, canal bank}
          b_rST.R^c = O × A,
          b_rST.R^i = 𝔎 × b_rST,
          b_rST.R^o = b_rST × 𝔎
        )

b_sST ≜ (A, O, R^c, R^i, R^o)                    // bank(storage)
      = ( b_sST.A  = {storage, container, place, organization},
          b_sST.O  = {information bank, human resource bank, blood bank}
          b_sST.R^c = O × A,
          b_sST.R^i = 𝔎 × b_sST,
          b_sST.R^o = b_sST × 𝔎
        )
```

Fig. 1. Formal and distinguished concepts derived from the word “bank”

Concept algebra provides a set of 8 relational and 9 compositional operations on abstract concepts as summarized in Table 1. Detailed definitions of operations defined in concept algebra may be referred to [14]. A Cognitive Learning Engine (CLE), known as the "CPU" of cCs, is under developing in my lab on the basis of concept algebra, which implements the basic and advanced cognitive computational operations of concepts and knowledge for cCs as outlined in Eq. 1b.

Additional concept operations may be introduced in order to reveal the underpinning mechanisms of learning and natural language comprehension. One of the advanced operations in concept algebra for knowledge processing is known as knowledge differential, which can be formalized in concept algebra as follows.

Definition 7. *Knowledge differential*, *dK/dt*, is an eliciting operation on a set of knowledge K represented by a set of concepts over time that recalls new concepts learnt during a given period t_1 through t_2, i.e.:

$$\begin{aligned} \frac{dK}{dt} &\triangleq \frac{d(OAR)}{dt} \\ &= OAR(t_2) - OAR(t_1) \\ &= OAR.C(t_2) \setminus OAR.C(t_1) \end{aligned} \tag{3}$$

where the set of concepts, $OAR.C(t_1)$, are existing concepts that have already been known at time point t_1.

Example 2. As given in Example 1, assume the following concepts, $OAR.C(t_1) = \{C_o\}$, are known at t_1, and the system's learning result at t_2 is $OAR.C(t_2) = \{C_o, C_r, C_s\}$. Then, a knowledge differential can be carried out using Eq. 3 as follows:

$$\begin{aligned} \frac{dK}{dt} &\triangleq \frac{d(OAR)}{dt} \\ &= OAR.C(t_2) \setminus OAR.C(t_1) \\ &= \{C_o, C_r, C_s\} \setminus \{C_o\} \\ &= \{C_r, C_s\} \end{aligned}$$

Concept algebra provides a powerful denotational mathematical means for algebraic manipulations of abstract concepts. Concept algebra can be used to model, specify, and manipulate generic "*to be*" type problems, particularly system architectures, knowledge bases, and detail-level system designs, in cognitive informatics, intelligence science, computational intelligence, computing science, software science, and knowledge science. The work in this area may also lead to a fundamental solution to computational linguistics, Computing with Natural Language (CNL), and Computing with Words (CWW) [34, 35].

4 Applications of Cognitive Informatics and Denotational Mathematics in Brain Informatics

This section introduces the notion of brain informatics as developed by Zhong and his colleagues [36]. A functional and logical reference model of the brain and a set of cognitive processes of the mind are systematically developed towards the exploration of the theoretical framework of brain informatics. The current methodologies for brain studies are reviewed and their strengths and weaknesses are analyzed.

Definition 8. *Brain informatics* (BI) is a joint field of brain and information sciences that studies the information processing mechanisms of the brain by computing and medical imagination technologies.

A variety of life functions and their cognitive processes have been identified in cognitive informatics, neuropsychology, cognitive science, and neurophilosophy. Based on the advances of research in cognitive informatics and related fields, a Layered Reference Model of the Brain (LRMB) is developed by Wang and his colleagues [28]. The LRMB model explains the functional mechanisms and cognitive processes of the natural and artificial brains with 43 cognitive processes at seven layers. LRMB elicits the core and highly recurrent cognitive processes from a huge variety of life functions, which may shed light on the study of the fundamental mechanisms and interactions of complicated mental processes as well as of cognitive systems, particularly the relationships and interactions between the inherited and the acquired life functions as well as those of the subconscious and conscious cognitive processes. Any everyday life function or behavior, such as reading or driving, is a concurrent combination of part or all of the 43 fundamental cognitive processes according to LRMB.

The basic methodologies in CI and BI are: a) logic (formal and mathematical) modeling and reasoning; b) empirical introspection; c) experiments (particularly abductive observations on brain patients); and d) using high technologies particularly brain imaging technologies. The central roles of formal logical and functional modeling for BI have been demonstrated in Sections 2 and 3 by CI, αI, and denotational mathematics. The advantage and disadvantaged of the latest methodologies of brain imaging are analyzed in the following subsections.

Modern brain imaging technologies such as EEG, fMRI, MEG, and PET are illustrated as shown in Fig. 2. Although many promising results on cognitive functions of the brain have been derived by brain imaging studies in cognitive tests and neurobiology, they are limited to simple cognitive functions compared with the entire framework of the brain as revealed in LRMB. Moreover, there is a lack of a systematic knowledge about what roles particular types of neurons may play in complex cognitive functions such as learning and memorization, because neuroimages cannot pinpoint to detailed relationships between structures and functions in the brain.

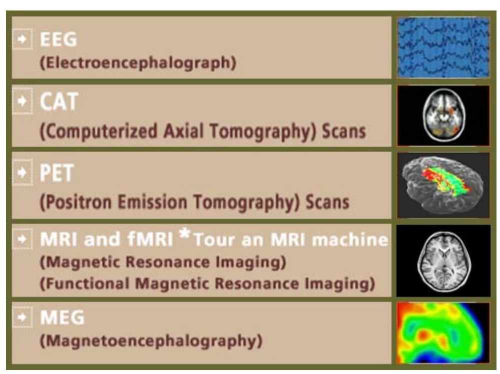

Fig. 2. Major imaging technologies in brain studies

The limitations of current brain imaging technologies such as PET and fMRI towards understanding the functions of the brain may be equivalent to the problem to exam the functions of a computer by looking at its layout and locations where they are active using imaging technologies. It is well recognized that without understanding the logical and functional models and mechanisms of the CPU as shown in Fig. 3, nobody can explain the functions of it by fine pictures of the intricate interconnections of millions of transistors (gates). Further, it would be more confusing because the control unit (CU) and arithmetic and logic unit (ALU) of the CPU and its buses are always active for almost all different kind of operations. So do unfortunately, brain science and neurobiology. Without a rational guide to the high-level life functions and cognitive processes as shown in the LRMB reference model, nobody may pinpoint rational functional relationship between a brain image and a specific behaviour such as an action of learning and its effect in memory, a recall of a particular knowledge retained in long-term memory, and a mapping of the same mental object from short-term memory to long-term memory.

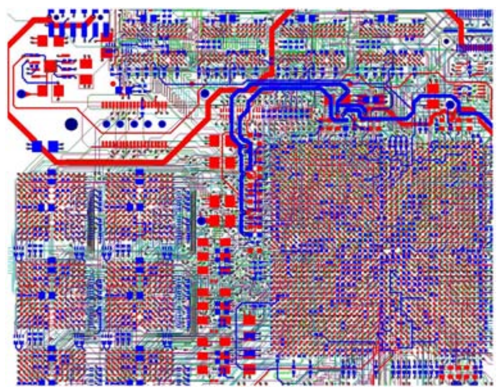

Fig. 3. The layout of a CPU

The above case study indicates that neuroscience theories and artificial intelligence technologies toward the brain have been studied at almost separate levels so far in biophysics, neurology, cognitive science, and computational/artificial intelligence. However, a synergic model as that of LRMB that maps the architectures and functions of the brain crossing individual disciplines is necessary to explain the complexity and underpinning mechanisms of the brain. This coherent approach will leads to the development of novel engineering applications of CI, αI, DM, CC, and BI, such as cognitive computers, artificial brains, cognitive robots, and cognitive software agents, which mimic the natural intelligence of the brain based on the theories and denotational mathematical means developed in cognitive informatics and abstract intelligence.

5 Conclusions

Cognitive Informatics (CI) has been described as a transdisciplinary enquiry of computer science, information sciences, cognitive science, and intelligence science that investigates into the internal information processing mechanisms and processes of the brain and natural intelligence, as well as their engineering applications in cognitive computing. *Brain informatics* (BI) has been introduced as a joint field of brain and information sciences that studies the information processing mechanisms of the brain by computing and medical imagination technologies.

This paper has presented some of the theoretical foundations of brain informatics developed in cognitive informatics, abstract intelligence, and denotational mathematics. In this paper, cognitive informatics as the science of abstract intelligence and cognitive computing has been briefly introduced. A set of denotational mathematics, particularly concept algebra, has been elaborated in order to enhance the fundamental theories and mathematical means for cognitive informatics, brain Informatics, and computational intelligence. Applications of cognitive informatics and denotational mathematics in brain informatics and cognitive computing are demonstrated based on the Layered Reference Model of the Brain (LRMB) and a set of cognitive processes of the mind towards the exploration of the theoretical framework of brain informatics.

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